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Remarks concerning the freeness problem over morphism and matrix semigroups. (English summary)
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The Open Question 1 in [J. Cassaigne and F. Nicolas, RAIRO Theor. Inform. Appl. **46** (2012), no. 3, 355–399; MR2981675] asked if the (k-)freeness problem of $\hom(\Sigma^\star)$ for a finite alphabet Σ with $|\Sigma| \geq 1$ is decidable. The (k-)freeness problem of a semigroup S with a recursive underlying set is whether or not a given morphism $\varphi\colon \Gamma^+ \to S$ is injective for a given finite alphabet Γ (with $|\Gamma| = k$). It is proved in this paper that the (14-)freeness problem of $\hom(\Sigma^\star)$ is undecidable when $|\Sigma| \geq 3$.

Also, the Open Question 5 in [op. cit.] asked if there exists a commutative semiring D with a recursive underlying set such that the 1-freeness problem of $\mathbb{M}_{2\times 2}(D)$ is decidable but the general freeness problem of $\mathbb{M}_{2\times 2}(D)$ is undecidable. This problem is solved in this paper by showing that the semiring $\mathbb{N} \times \mathbb{N}$ has the above property: the 1-freeness problem of $\mathbb{M}_{2\times 2}(\mathbb{N} \times \mathbb{N})$ is decidable but its 13-freeness problem is undecidable. It is interesting to note that the above question is still open when D is taken to be a field. *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.